New way for simulation of transient natural convection heat transfer

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A method for improving numerical solution of transient natural convection heat transfer in enclosures is proposed, where temperature, a stream function, and vorticity are decomposed into Fourier components of a body-fitted curvilinear coordinate. Using addition formulas of trigonometric functions, the equations of motion, energy, and continuity can also be separated into Fourier series. This reduces the number of variables by one and leads to reduction of the numerical computation time.

As an example, given is a seven-terms numerical solution for a Grashof number of 19,600 in case of air in a circular cylinder.

Keywords: *natural convection; transient heat transfer; numerical simulation*

Introduction

Natural convection heat transfer is essentially different from forced convection heat transfer in the sense that the flows of natural convection are forced neither by an externally applied pressure gradient nor by only moving boundaries. Mathematically, the flow field of natural convection is coupled to the temperature field, the variation of which in space will lead to a driving force, a buoyancy force, in connection with gravitational acceleration. In a mathematical viewpoint, fields of natural convection are classified into two categories: flow fields around a body or bodies and flow fields in enclosures. Of these, the natural convection flow in an enclosure is an essentially recirculating flow. Therefore, for these flows, treatment of nonlinear convective terms in the motion equations and the energy equation is quite significant. For example, natural convection in a rectangular cavity can be treated analytically in the conduction limit (at a low Rayleigh number), the high Rayleigh number limit (boundary layer regime), the tall enclosure limit, or the shallow enclosure limit[1] and has been analyzed numerically by applying finite difference schemes to the governing equations.[2-4] This finite difference approximation in space requires a larger number of grids in space as Rayleigh number increases, which requires a larger computation time, especially in solving simultaneous equations constructed for transient or unsteady natural convection heat transfer.

In this paper, I propose a method for improving numerical computation, where one independent variable in space is separated even for the nonlinear convective terms, thus the number of independent variables is reduced by one, and the numerical computation time is greatly reduced.

Analysis

Mathematical model

Transient or unsteady laminar natural convection in a duct of a constant cross section placed horizontally is considered. Mean temperature difference in fluid is assumed to be small, and slight expansion or compression of fluid in a horizontal direction may be allowed so that the thermodynamic state is essentially isobaric. Under this assumption, the Boussinesq approximation for the variation of density holds. All the more, the following are assumed in forming the governing equations:

(1) The fluid is Newtonian.
(2) Thermal properties are constant except in the body force.
(3) The flow and temperature field is essentially two dimensional.
(4) Effects of compressibility and viscous dissipation are negligible.

Under these assumptions, the governing equations derived from the equation of motion, the equation of continuity, and the equation of energy become

$$\rho\left(\frac{\partial}{\partial t}+\mathbf{V}\cdot\nabla\right)\text{curl }\mathbf{V}=\mu\Delta(\text{curl }\mathbf{V})+\nabla\rho\times\mathbf{g}$$
$$=\mu\Delta(\textbf{curl }\mathbf{V})-\rho\beta\nabla T\times\mathbf{g} \tag{1}$$

$$\nabla\cdot\mathbf{V}=0 \tag{2}$$

$$\rho c_P\left(\frac{\partial}{\partial t}+\mathbf{V}\cdot\nabla\right)T=k\Delta T \tag{3}$$

respectively. Let x, y, z be a right-handed Cartesian coordinate system so that the z axis is horizontal and normal to the flow plane and the y axis is vertical and upward. Then from assumption 3, x and y components of curl $\mathbf{V}$ and $\nabla T\times\mathbf{g}$ vanish identically. Let ζ denote the nonzero z component of curl $\mathbf{V}$. Then Equation 1 is rewritten

$$\rho\left(\frac{\partial}{\partial t}+\mathbf{V}\cdot\nabla\right)\zeta=\mu\Delta\zeta-\rho\beta(\nabla T\times\mathbf{g})_z$$
$$=\mu\Delta\zeta+\rho g\beta(\nabla T)_x \tag{4}$$

where the subscripts z and x mean the z component and the x component, respectively. The velocity $\mathbf{V}$ and the vorticity ζ are coupled with a two-dimensional stream function ψ through

$$\mathbf{V}=\text{curl}(0,0,\psi) \tag{5}$$

$$\zeta=-\Delta\psi \tag{6}$$

Thus the system of equations to be solved consists of Equations 3–6, supplemented with boundary conditions and initial conditions.

Proposal for the method of separation of one independent variable

Hereafter, the domain of the flow field is assumed to be simply connected or doubly connected. Let ξ and η be suitably selected body-fitted spatial coordinates in the xy plane so that without loss of generality, the domain is mapped into $0\leqslant\xi\leqslant1$ and $-\pi\leqslant\eta\leqslant\pi$, where for a doubly connected region, the boundaries correspond to $\xi=0$ and $\xi=1$, and for a simply connected region, the boundary corresponds to $\xi=1$ only or $\xi=0$ and $\xi=1$. In the case that only $\xi=1$ represents the boundary of a simply connected region, $\xi=0$ represents a line of

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finite length inside the boundary or reduces to a single point on or inside the boundary; this depends on the coordinate system used. The points for which $\eta=\pi$ may correspond to the points for which $\eta=-\pi$. Let ζ, ψ, and T be decomposed into Fourier series in η as

$$\zeta=\sum_{m=0}^{\infty}\zeta_{cm}(\xi,t)\cos m\eta+\sum_{m=1}^{\infty}\zeta_{sm}(\xi,t)\sin m\eta \tag{7}$$

$$\psi=\sum_{m=0}^{\infty}\psi_{cm}(\xi,t)\cos m\eta+\sum_{m=1}^{\infty}\psi_{sm}(\xi,t)\sin m\eta \tag{8}$$

$$T=\sum_{m=0}^{\infty}T_{cm}(\xi,t)\cos m\eta+\sum_{m=1}^{\infty}T_{sm}(\xi,t)\sin m\eta \tag{9}$$

The quantities $\nabla\zeta$, $\Delta\zeta$, ∇T, ΔT, and curl$(0,0,\psi)$ are linear with respect to ζ, ζ, T, T, and ψ, respectively. In a general curvilinear coordinate system, the physical components of $\nabla\zeta$, ∇T, and curl$(0,0,\psi)$ are composed of the sum of ζ or its partial derivative with respect to space, T or its partial derivative, or ψ or its partial derivative, all multiplied by metric tensors, its partial derivatives, or their related functions. These multipliers can be expanded into Fourier series in η (except singular point(s) if any); thus using the addition formulas of trigonometric functions, $\nabla\zeta$, ∇T, and curl$(0,0,\psi)$ can be decomposed into Fourier series in η. Likewise $\Delta\zeta$, $\Delta\psi$, and $(\nabla T)_x$ $[\equiv(\partial T/\partial\xi)(\partial\xi/\partial x)+(\partial T/\partial\eta)(\partial\eta/\partial x)]$ can be decomposed into Fourier series in η. Since $(\mathbf{V}\cdot\nabla)\zeta=\mathbf{V}\cdot(\nabla\zeta)$, and since $\mathbf{V}$ and $\nabla\zeta$ can be expressed as a Fourier series in η, $(\mathbf{V}\cdot\nabla)\zeta$ can also be decomposed into a Fourier series in η, using the addition formulas of trigonometric functions. A similar situation holds for $(\mathbf{V}\cdot\nabla)T$. Thus substituting Equations 7, 8, and 9 into Equations 3–6 constitutes a system of infinite simultaneous partial differential equations for ζ_{cm}s, ζ_{sm}s, ψ_{cm}s, ψ_{sm}s, T_{cm}s, and T_{sm}s.

Boundary conditions

Equation 9 is compatible with the following thermal boundary conditions. Along the boundary $\xi=1$, either temperature or heat flux through it is prescribed (the Dirichlet type condition or the Neumann type condition). Also along the boundary $\xi=0$ if it exists, independent of the type of the condition imposed on $\xi=1$, either the Dirichlet type condition or the Neumann type condition is prescribed.

In the case that $\xi=0$ does not constitute a boundary in the physical plane for a simply connected region, the condition derived from the fact that the temperature field belongs to a C^2-class field there will be supplemented. Given a temperature distribution function $T(\xi_i,\eta,t)$ along the boundary $\xi=\xi_i$ for the Dirichlet type condition, the values of the function $T_{cm}(\xi_i,t)$ and $T_{sm}(\xi_i,t)$ can be determined as the Fourier coefficients of the function $T(\xi_i,\eta,t)$. Given the normal component of temperature gradient $\mathbf{n}\cdot\nabla T$ along the boundary for the Neumann type condition,

$$\mathbf{n}\cdot\nabla T=\frac{\partial T}{\partial\xi}(\mathbf{n}\cdot\nabla)\xi+\frac{\partial T}{\partial\eta}(\mathbf{n}\cdot\nabla)\eta \tag{10}$$

where $\mathbf{n}$ is an outward unit normal and, using Equation 9, the right-hand side of this equation can be estimated. The terms $(\mathbf{n}\cdot\nabla)\xi$ and $(\mathbf{n}\cdot\nabla)\eta$ can be expanded into Fourier series in η; thus the right-hand side of Equation 10 can be decomposed into Fourier series in η, using the addition formulas of trigonometric functions. Therefore, by expanding the left-hand side of Equation 10 into Fourier series in η and by equating like components of sine and cosine functions, relations at $\xi=\xi_i$, which are linear in $\partial T_{cm}/\partial\xi$, $\partial T_{sm}/\partial\xi$, T_{cm}, and T_{sm}, can be obtained.

As for the boundary conditions for velocity, no-slip conditions at the boundary apply. That is, along the boundaries

$$\psi=\text{constant} \tag{11}$$

$$\mathbf{n}\cdot\nabla\psi=0 \tag{12}$$

Equations 11 and 12 can be decomposed into Fourier series in η along the boundary $\xi=$constant. Moreover, in the case that $\xi=0$ does not constitute a physical boundary for a simply connected region, the condition that the stream function ψ and the vorticity ζ are C^2-class functions there will be supplemented.

Numerical solution procedure

Here it is assumed initial conditions for ζ, ψ, and T can be expanded into the forms of Equations 7–9. In the case that Equations 7–9 converge uniformly or asymptotically, truncation of terms possessing higher Fourier components in Equations 7–9 and keeping only the corresponding first few Fourier components in Equations 3–6 gives rise to a system of simultaneous nonlinear partial differential equations for a finite number of unknowns. To get a solution of this system for transient or unsteady natural convection heat transfer, it is convenient to introduce a forward difference formula for partial time derivatives and possibly any type of finite difference formulas for spatial derivatives. Implicit schemes are suggested for specifying the time when each term except local acceleration terms possessing the operator $\partial/\partial t$ is estimated, that is, in equations corresponding to the $\cos m\eta$ or $\sin m\eta$ component,

Notation

a	Radius at a circular cross section
c_p	Specific heat at constant pressure
g	Gravitational acceleration
$\mathbf{g}$	Gravitational acceleration vector
Gr	Grashof number defined in Eq. (14)
h	Distance between neighboring two grid points
k	Thermal conductivity
$\mathbf{n}$	Outward unit normal
Pr	Prandtl number
r	Radial coordinate in a cylindrical polar coordinate system
T	Temperature
t	Time
T_c	Temperature at the origin
T_i	Initial temperature
T_w	Wall temperature
U_0	Reference velocity
$\mathbf{V}$	Velocity vector
V_c	y component of velocity at the origin
x	Coordinate in a Cartesian coordinate system
y	Coordinate in a Cartesian coordinate system
z	Coordinate in a Cartesian coordinate system
β	Coefficient of thermal expansion
δt	Time increment
ζ	Vorticity
η	Coordinate in a body-fitted curvilinear coordinate system
θ	Tangential coordinate in a cylindrical polar coordinate system
μ	Viscosity of fluid
ν	Kinematic viscosity of fluid
ξ	Coordinate in a body-fitted curvilinear coordinate system
ρ	Density of fluid
ψ	Stream function
Δ	Laplacian operator
∇	Gradient operator

local acceleration terms $\partial\phi/\partial t$, where ϕ denotes ζ_{cm}, ζ_{sm}, T_{cm}, or T_{sm}, are evaluated as

$$\frac{\partial}{\partial t}\phi(\xi,t)\approx\frac{1}{\delta t}\{\phi(\xi,t+\delta t)-\phi(\xi,t)\} \tag{13}$$

where t is the current time. Other terms that are linear functions of unknowns should be evaluated at time $t+\delta t$ if and only if the second suffices of the unknowns are the same as m, whereas all other remaining terms including nonlinear terms are to be evaluated at time t. The same applies to the boundary conditions. These processes constitute a system of linear simultaneous equations for unknowns over the space ξ at time $t+\delta t$. All the more, the resultant linear equations for different periods in η become independent of one another and can be solved with consuming shorter computation time. Thus the system of equations can be solved in advancing time step by step with initial conditions.

An example of the analysis

Flow configuration

As an example, the current proposed method is applied to a transient natural convection heat transfer problem, that is, Newtonian fluid enclosed in a circular cylinder of radius a placed horizontally is assumed to be initially at rest ($t<0$) under isothermal conditions, and a uniform step change of wall temperature takes place at $t=0$, after which the temperature at the wall will be held constant. For convenience, initial increase of wall temperature is assumed.

Formulation of the problem

Let Gr denote a Grashof number, which is defined as

$$\mathrm{Gr}=\frac{g\beta a^3}{\nu^2}(T_w-T_i) \tag{14}$$

where T_i is the initial temperature of the fluid, and T_w is the constant temperature of the wall for $t\geqslant 0$. Hereafter for simplicity, coordinates, velocity, time, a stream function, and vorticity are nondimensionalized with respect to a, U_0, a/U_0, aU_0, and U_0/a, respectively, where U_0 is a reference velocity and is defined as

$$U_0=\frac{\nu}{a}\mathrm{Gr}^{0.5}$$

Temperature is nondimensionalized as

$$\frac{(T_w-T)}{(T_w-T_i)}$$

and hereafter T stands for this nondimensionalized temperature. To describe the heat and fluid flow, a cylindrical polar coordinate system (r,θ,z) is used as in usual orientation, the origin being located at the center of the circular cross section. Then, Equations 3, 4, and 6 become

$$\left(\frac{\partial}{\partial t}+\frac{1}{r}\frac{\partial\psi}{\partial\theta}\frac{\partial}{\partial r}-\frac{1}{r}\frac{\partial\psi}{\partial r}\frac{\partial}{\partial\theta}\right)T=\frac{1}{\mathrm{Pr}\cdot\mathrm{Gr}^{0.5}}\Delta T \tag{15}$$

$$\left(\frac{\partial}{\partial t}+\frac{1}{r}\frac{\partial\psi}{\partial\theta}\frac{\partial}{\partial r}-\frac{1}{r}\frac{\partial\psi}{\partial r}\frac{\partial}{\partial\theta}\right)\zeta=\frac{1}{\mathrm{Gr}^{0.5}}\Delta\zeta-\left(\cos\theta\frac{\partial}{\partial r}-\frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\right)T \tag{16}$$

$$\zeta=-\Delta\psi \tag{17}$$

respectively, where Δ is a two-dimensional Laplacian operator defined as

$$\Delta=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} \tag{18}$$

The coordinates r and θ correspond to ξ and η, respectively, in the general notation used previously. Initial conditions at $t=0$ are

$$\begin{aligned}&T=0 \quad \text{for } |r|<1\\ &\psi=0 \quad \text{for } |r|\leqslant 1\end{aligned} \tag{19}$$

Boundary conditions at $t\geqslant 0$ are

$$\begin{aligned}&T=0 \quad \text{for } r=1\\ &\psi=0 \quad \text{for } r=1\\ &\frac{\partial\psi}{\partial r}=0 \quad \text{for } r=1\end{aligned} \tag{20}$$

Under these initial and boundary conditions, Equations 15–17 have the following formal solutions:

$$T=\sum_{m=0}^{\infty}T_{2m}(r,t)\cos 2m\theta+\sum_{m=0}^{\infty}T_{2m+1}(r,t)\sin(2m+1)\theta \tag{21}$$

$$\psi=\sum_{m=0}^{\infty}\psi_{2m+1}(r,t)\cos(2m+1)\theta+\sum_{m=1}^{\infty}\psi_{2m}(r,t)\sin 2m\theta \tag{22}$$

$$\zeta=\sum_{m=0}^{\infty}\zeta_{2m+1}(r,t)\cos(2m+1)\theta+\sum_{m=1}^{\infty}\zeta_{2m}(r,t)\sin 2m\theta \tag{23}$$

Substituting Equations 21–23 into Equations 15–17 and decomposing into Fourier components produces a system of nonlinear partial differential equations for T_ms, ψ_ms, and ζ_ms; the $\cos(0\theta)$ component of Equation 15 is given as an example:

$$\frac{\partial}{\partial t}T_0-\frac{\partial}{\partial r}\sum_{m=0}^{\infty}\frac{2m+1}{2r}\psi_{2m+1}T_{2m+1}+\frac{\partial}{\partial r}\sum_{m=1}^{\infty}\frac{m}{r}\psi_{2m}T_{2m}=\frac{1}{\mathrm{Pr}\cdot\mathrm{Gr}^{0.5}}\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)T_0 \tag{24}$$

The boundary conditions (Equation 20) at $r=1$ for $t\geqslant 0$ become

$$\begin{aligned}&T_m=0 \quad (m\geqslant 0)\\ &\psi_m=0 \quad (m\geqslant 1)\end{aligned} \tag{25}$$

$$\frac{\partial}{\partial r}\psi_m=0 \quad (m\geqslant 1) \tag{26}$$

The initial conditions (Equation 19) at $t=0$ become

$$\begin{aligned}&T_0=1 \quad \text{for } |r|<1\\ &T_m=0 \quad \text{for } |r|\leqslant 1, \quad (m\geqslant 1)\\ &\psi_m=\zeta_m=0 \text{for } |r|\leqslant 1, \quad (m\geqslant 1)\end{aligned} \tag{27}$$

Supplementary conditions at $r=0$ derived from Equations 15–17 are

$$\begin{aligned}&\frac{\partial}{\partial r}T_0=0\\ &T_m=0 \quad (m\geqslant 1)\\ &\psi_m=\zeta_m=0 \quad (m\geqslant 1)\end{aligned} \tag{28}$$

Under the conditions of Equations 25–28, the functions T_m, ψ_m, and ζ_m possess the following nature:

$$T_m\sim 0(\mathrm{Gr}^m) \tag{29}$$

$$\psi_m\sim 0(\mathrm{Gr}^{m-0.5}) \tag{30}$$

$$\zeta_m\sim 0(\mathrm{Gr}^{m-0.5}) \tag{31}$$

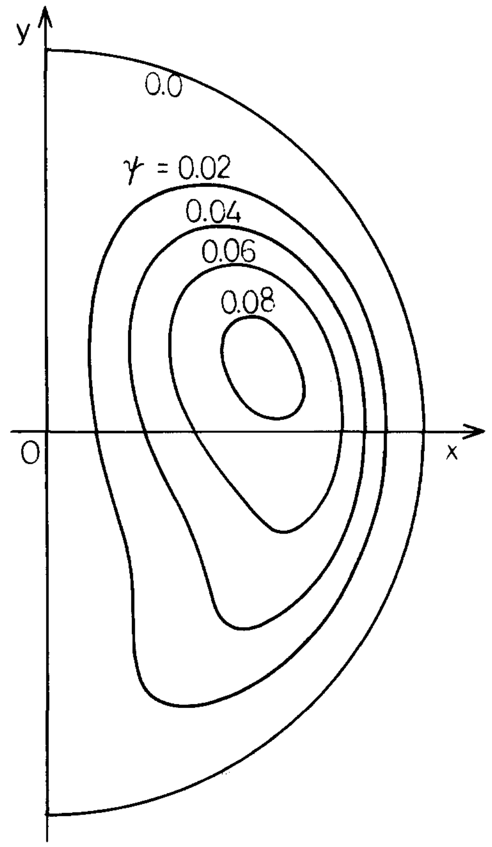

Figure 1 Streamlines in the right half section for transient natural convection at $t/Gr^{0.5}=0.0508$ (Gr=19,600, Pr=0.72). The pattern in the left half section is the reflection of that shown with respect to the y axis, although the stream functions are opposite in sign

as Gr tends to zero while $t/Gr^{0.5}$ remains finite. Thus using equally spaced grids in the r direction, the system of equations for T_ms, ψ_ms, and ζ_ms can be solved numerically according to the solution procedure previously mentioned, where Equation 26 is equivalent with good accuracy to

$$\zeta_m(1-h,t)\approx -\frac{2}{h^2}\psi_m(1-h,t) \tag{32}$$

and h is the distance between two adjacent grid points.

Numerical results

In the following numerical example, only the first seven functions in Equations 21–23 are retained, and higher components are neglected; that is, only the functions $T_0 \sim T_6$, $\psi_1 \sim \psi_7$, and $\zeta_1 \sim \zeta_7$ are adopted, and only the equation corresponding to the Fourier components of these functions are used.

Figures 1 and 2 show patterns of streamlines and isotherms in the right half section for Pr = 0.72 (Air), Gr = 19,600, $t/Gr^{0.5} = 0.0508$ ($h = 0.04$). Figure 3 shows the variation of the temperature at the origin, T_c, with time for Pr = 0.72, Gr = 19,600. Figure 4 shows the downward flow velocity at the origin, $-V_c$, with time for Pr = 0.72, Gr = 19,600.

Discussion

Low Grashof number limit

In the low Grashof number limit, both in Equations 3 and 4, nonlinear convective acceleration terms can be neglected to give

$$T=\sum_{m=1}^{\infty}\frac{2}{\beta_m J_1(\beta_m)}J_0(\beta_m r)\exp(-\beta_m^2 t/(\mathrm{Pr}\cdot \mathrm{Gr}^{0.5})) \tag{33}$$

$$\begin{aligned}\psi = -\frac{2\,\mathrm{Pr}\cdot\mathrm{Gr}^{0.5}\cos\theta}{1-\mathrm{Pr}}\Bigg[&\sum_{m=1}^{\infty}\frac{1}{\beta_m^4 J_1(\beta_m)}\exp(-\beta_m^2 t/(\mathrm{Pr}\cdot\mathrm{Gr}^{0.5}))\\ &\times\Bigg\{J_1(\beta_m r)+\frac{J_1(\beta_m)J_0(\beta_m/\mathrm{Pr}^{0.5})}{J_2(\beta_m/\mathrm{Pr}^{0.5})}r\\ &-\frac{2\,\mathrm{Pr}^{0.5}J_1(\beta_m)}{\beta_m J_2(\beta_m/\mathrm{Pr}^{0.5})}J_1(\beta_m r/\mathrm{Pr}^{0.5})\Bigg\}\\ &+\sum_{m=1}^{\infty}A_m\{J_1(\gamma_m r)-J_1(\gamma_m)r\}\exp(-\gamma_m^2 t/\mathrm{Gr}^{0.5})\Bigg]\end{aligned} \tag{34}$$

$$A_m\equiv\frac{2}{J_1^2(\gamma_m)}\sum_{n=1}^{\infty}\frac{1}{\beta_n^4}\left\{\frac{\gamma_m J_0(\gamma_m)}{\gamma_m^2-\beta_n^2}-\frac{2J_1(\gamma_m)}{\gamma_m^2-\beta_n^2/\mathrm{Pr}}\right\} \tag{35}$$

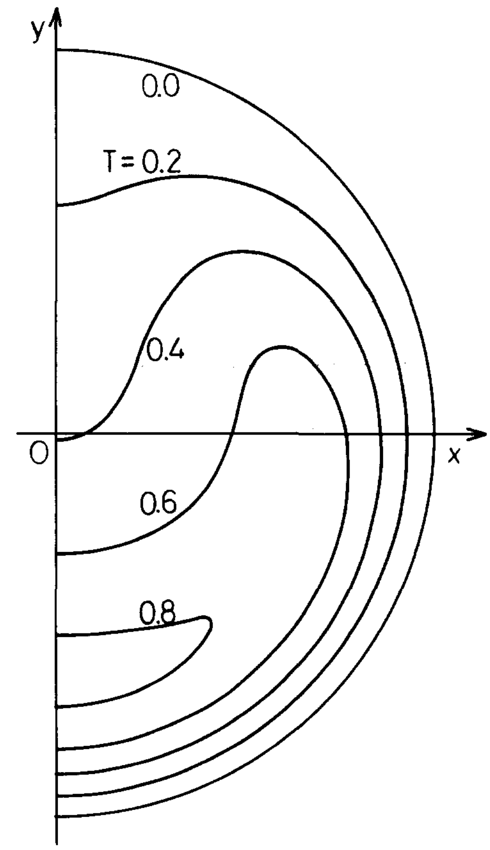

Figure 2 Isobars in the right half section for transient natural convection at $t/Gr^{0.5}=0.0508$ (Gr=19,600, Pr=0.72). The pattern in the left half section is the reflection of that shown with respect to the y axis

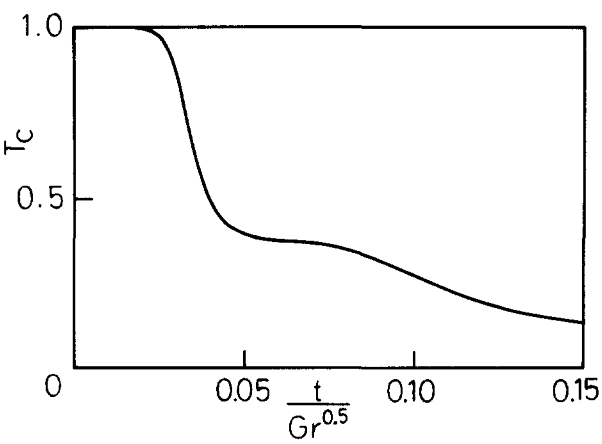

Figure 3 The variation of the temperature at the origin, T_c, with time (Gr=19,600, Pr=0.72)

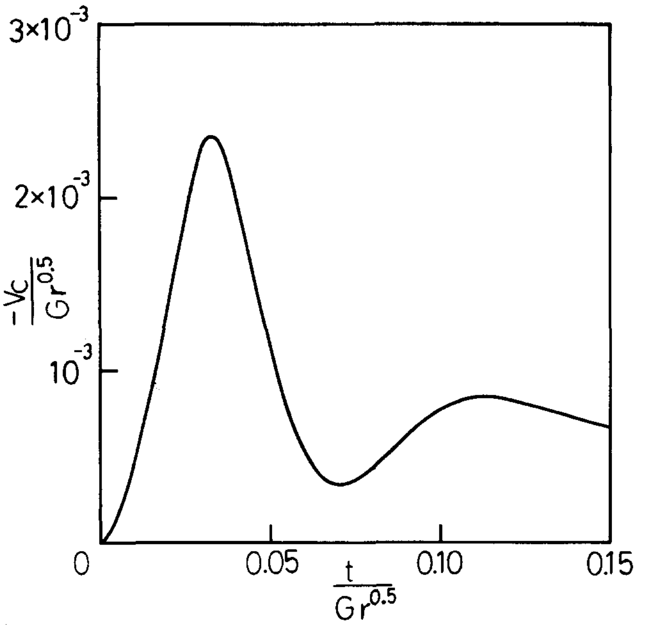

Figure 4 The variation of the downward flow velocity at the origin, $-V_c$, with time (Gr=19,600, Pr=0.72)

where J_0, J_1, and J_2 are the Bessel functions of orders 0, 1, and 2, respectively; β_m and γ_m are the positive mth zeros of $J_0(x)$ and $J_2(x)$, respectively. It is also assumed $Pr \neq 1$ and $Pr \neq \beta_n^2/\gamma_m^2$ for any integers m and n, that is, $J_2(\beta_m/Pr^{0.5}) \neq 0$. Equations 33 and 34 are good approximations to the numerical solution for a set of Equations 15–17 up to a Grashof number of approximately 100 (Pr = 0.72).

Start-up of heat and fluid flow

Since all the boundaries are stationary, the vorticity is continuous at $t = 0$ with respect to time. Therefore, even if the initial temperature gradient at the wall is mathematically infinite, the current system of equations can be solved stably by the proposed implicit schemes, using finite difference methods for spatial derivatives, although the numerical solutions obtained in this way possess a slight error only for a short while after the start-up of flow. For precisely estimating the start-up of flow in a short interval, Equations 33 and 34 can be used.

High Grashof number flow

Use of the first seven Fourier components produces good numerical solutions at least up to a Grashof number of approximately 40,000 in the sense that terms of higher Fourier components are almost small compared with those of the lower components. If the Grashof number gets greater than 10^5, more components will become necessary.

Conclusion

A method for improving numerical solution of transient or unsteady natural convection heat transfer enclosed in duct wall(s) is proposed, where use is made of Fourier decomposition for temperature, the stream function, and vorticity. As an example, given is a seven-terms numerical solution for a Grashof number of 19 600 in case of air in a circular cylinder facing to a step change of wall temperature.

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